B Tech.(1st Sem.) MATH-I
DEPARTMENT OF MATHEMATICS, VSSUT, BURLA
MID SEMESTER EXAMINATION

SUBJECT – MATHEMATICS - I

Full Marks: 20 Time-2 hours

Answer Q.No.1 which is compulsory and any three questions out of the rest questions.

The figures in right hand margin indicate marks.

1.Answer all parts of this question. **[1 x 5]**

a) Define rank of a matrix.

b) Are the row vectors [1 1 1], [0 0 1] and [1 1 0] linear independent?

c) Find the derived set of the set of all integers Z.

d) Give an example of a set which is both open & closed.

e) Is the set {1/n, n ∈ N} is closed. Justify ?

2. (a) Using Gauss Elimination method solve **[2.5]**

$$2X + 3Y + Z - 11W = 1$$
$$5X - 2Y + 5Z - 4W = 5$$
$$X - Y + 3Z - 3W = 3$$
$$3X + 4Y - 7Z + 2W = -7$$

(b) Find the rank of the matrix $A=\begin{pmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{pmatrix}$ **[2.5]**

3. (a) Is the set of all vectors (v_1, v_2, v_3) in R^3 such that $2v_2+3v_3 = 0$ form a vector space? If yes find the dimension and find a basis. **[2.5]**

(b) Find the inverse of the following matrix by Gauss-Jordan elimination method. **[2.5]**

$$A=\begin{pmatrix} 4 & -1 & 5 \\ 15 & 1 & -5 \\ 5 & -4 & 9 \end{pmatrix}$$

4. (a) Find the rank of the following matrix using determinant

$$A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$ **[2.5]**

(b) Show that the set {1, 1/2, -1/2, 1/3, -1/3, ... } is neither open or closed. **[2.5]**

5. (a) Using ε- δ technique find the right hand limit of $\frac{1}{(1+e^{-1/x})}$ as $x \to 0$. **[2.5]**

(b) Test the continuity of **[2.5]**

$$f(x)=\begin{cases} -x^2, & x \le 0 \\ 5x-4, & 0 < x < 1 \\ 4x^2-3x, & 1 < x < 2 \\ 3x+4, & x \ge 2 \end{cases}$$ at x = 0, 1, 2.

6.(a) Prove that union of arbitrary family of open sets is open. **[2.5]**

(b) Test discontinuity of the function f(x) = x – [x] at x = 3. **[2.5]**

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